ISIP Math Teacher Resource: Number Sense – Comparing Fractions

Resource at a Glance

Grade 4: Number Sense

Primary Skills Addressed:
- **Understanding fraction comparison**

Supporting Skills:
- **Developing an understanding of decimals**
- **Developing the connections between fractions and decimals**

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Teacher-Directed Lesson

Lesson Objective

Students should be able to compare fractions with the same denominators but different numerators. They will be given a procedural strategy, including drawing representations, to use when comparing.

Prerequisite Skills and Knowledge

- Understanding the concept of fractions
- Understanding the differences in meanings of the numerator and denominator

Materials and Resources

- Pencils and paper
- Copies of Practice Problems
- Teacher Resource 1 (optional)
- Copies of checklist (optional)
- Fraction charts (optional)

Additional Accommodations

Tier 2: Provide students with individual copies of the checklist or display it on a poster in the classroom for students to reference as they complete the problems. Provide additional support, including multiple examples and non-examples, to give students an opportunity to practice what they have learned. Students may need a reminder on the proper use of the greater than (>) and less than (<) symbols. Consider reviewing with whole numbers first before applying with fractions.

Tier 3: Students could be given a page of fractions charts that have common fractions already divided, such as into 2–12 pieces. By having access to this, students can easily create and compare visual models. Students may need a reminder on the proper use of the greater than (>) and less than (<) symbols. Consider reviewing with whole numbers first before applying with fractions.

Vocabulary

denominator – the number of equal-sized pieces that make up one whole

numerator – the number of equal-sized pieces that are selected from the whole

ISIP Math Teacher Resource: Number Sense – Comparing Fractions

Note: Words in bold are said aloud by the teacher.

Introduction

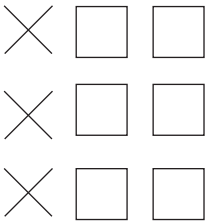
The objective of this lesson is to compare fractions by looking at the numerators and denominators to find which fraction is larger.

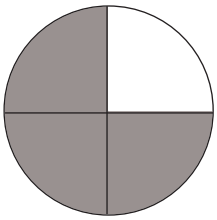
Let's start by reviewing the parts of a fraction. Display the definitions on Teacher Resource 1, or write them on the board, as shown below.

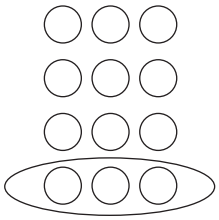
<u>denominator</u> – the number of equal-sized pieces that make up one whole

<u>numerator</u> – the number of equal-sized pieces that are selected from the whole

The denominator tells the total number of parts that make up the whole. The numerator is the number of parts that are counted from the whole. Display the models on Teacher Resource 1, or draw them on the board, as shown below.







In the first model, how many shapes are Xs? *(three)* **How many shapes are there total?** *(nine)* **Which number is the numerator?** *(3)* **What is the denominator?** *(9)* **So we would write this as the fraction three-ninths.** Write $\frac{3}{9}$ below the first model.

In the second model, how many pieces make up the whole? *(four)* **How many pieces are shaded?** *(three)* **Is three the numerator or the denominator?** *(numerator)* **What is the denominator?** *(4)* **What is the fraction?** *($\frac{3}{4}$)* Write $\frac{3}{4}$ below the second model.

I**n our third model, we want to write a fraction for the circled section. What is the numerator?** *(3)* **What is the denominator?** *(12)* **How would we write our fraction?** *($\frac{3}{12}$)* Write $\frac{3}{12}$ below the third model.

Model

Today we are going to talk about how to compare fractions and tell which fraction is greater. These fractions will have the same denominator but not the same numerator. Which part of the fraction is the denominator? *(the number on the bottom)* **What does the denominator represent?** *(the total amount of pieces in the whole)* **If fractions have the same denominator, they are cut into the same number of equal pieces. For example, look at these two models.**

Model (cont.)

Reveal these models on Teacher Resource 1, or draw them for students to see. **Twelve would be the denominator for both of the fractions because there are twelve total pieces in each model.**

How many pieces are shaded in the first model? *(2)* **How would we write this fraction?** *($\frac{2}{12}$)*
How many pieces are shaded in the second model? *(5)* **How would we write this fraction?** *($\frac{5}{12}$)*

By looking at the pictures, which model represents the bigger fraction? *(the one on the right)*
Why? *(five pieces are shaded compared to two and five is more than two)*

This means that five-twelfths is greater than two-twelfths. We would write this as $\frac{5}{12} > \frac{2}{12}$. Write $\frac{5}{12} > \frac{2}{12}$ for students to see. Reinforce the use of mathematical symbols. For some students with limited knowledge or understanding of these symbols, additional instruction or review may be needed.

So, if you have two fractions with the same denominator, the fraction with the larger numerator is the bigger number. We can remember these simple steps when the denominators are the same. Reveal the steps on Teacher Resource 1, or write them on the board.

For the same denominator:

- **pieces are the same size**
- **more pieces = greater amount**
- **larger numerator = larger fraction**

Let's try one together using these steps. Write $\frac{7}{9} ? \frac{3}{9}$ for students to see.

Since they have the same denominator we know the pieces are the same size. Next we look at the numerators. Which numerator is the greater number? *(7)* **What does this mean about how our two fractions compare to each other?** *(since 7 is greater than 3, $\frac{7}{9} > \frac{3}{9}$)*

So, when comparing fractions you want to look at both the numerator and denominator. If the denominator is the same, then there is the same number of parts in the whole. At that time, you can compare numerators. The fraction with the larger number of parts is the greater fraction.

Guided and Independent Practice

Make sure each student has a pencil and Practice Problems. Guide students in making comparisons for numbers 1–3, and have them complete 4–6 independently. Instruct students to determine which fraction is greater and fill in the expressions with < or > symbols. Ask students to think aloud during the discussion.

Answer Key

1. <
2. <
3. >
4. <
5. <
6. >

Evaluation/Feedback

Review independent practice problems with students. Ask a student volunteer to read a problem and then tell how he or she solved it. Make sure the student verbalizes in the way you verbalized during the teacher model. If the student struggles with an answer or explanation, guide him or her through the lesson's checklist.

Reteaching/Extensions

If students struggle to make comparisons between the abstract representations of fractions, draw models for some examples (fraction charts, pie charts, etc.) and connect them to the abstract by writing the fraction and expressions with the models.

Monitor students as they work and provide feedback when errors are made. Display the lesson's checklist for students to see when they are working on their practice problems. Students may need individual laminated copies of the checklist to mark while they are working through problems.

As an extension to the lesson, guide students through comparing fractions with unlike numerators and denominators which will need to be converted or simplified in order to make comparisons. If students demonstrate proficiency with the given problems, you may add more complex mixed numbers and improper fractions for them to compare.

Closure

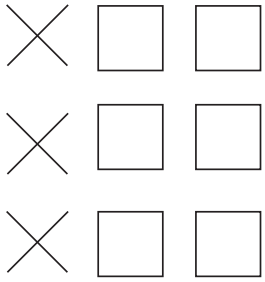
Remind students that the objective of the lesson was to learn to compare fractions and determine which ones are larger or smaller by comparing the numerators and denominators. Emphasize the use of fractions in everyday life, such as with food portions. Point out that many professionals, such as musicians, chefs, and carpenters, also use fractions and decimals. Encourage students to look for other ways fractions are used every day.

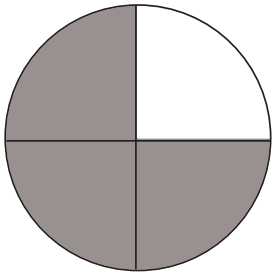
Teacher Resource 1

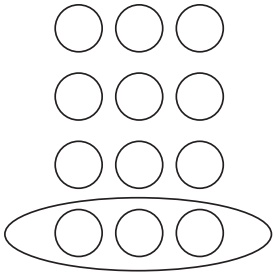
Parts of a fraction:

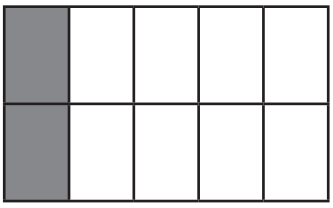
denominator – the number of equal-sized pieces that make up one whole

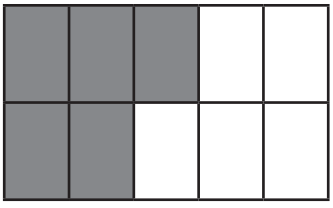
numerator – the number of equal-sized pieces that are selected from the whole











Comparing fractions:

For the same denominator:

- pieces are the same size
- more pieces = greater amount
- larger numerator = larger fraction

Number Sense – Comparing Fractions

Name: _____ **Date:** _____

Practice Problems

Determine which fraction is greater and fill in the expressions with < or > symbols.

1. $\frac{3}{5} \square \frac{4}{5}$

2. $\frac{5}{15} \square \frac{11}{15}$

3. $\frac{6}{9} \square \frac{6}{11}$

4. $\frac{7}{15} \square \frac{7}{8}$

5. $\frac{15}{4} \square \frac{15}{3}$

6. $\frac{12}{5} \square \frac{6}{5}$

Appendix

Integration of the Content and Research-Based Instructional Practice

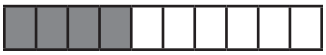
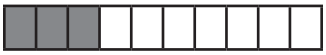
The Institute of Education Sciences (IES) recommends that interventions include recurring instruction that is explicit and systematic in design and delivery. The IES Practice Guide recommends that teachers regularly provide struggling students with models of unambiguous step-by-step problem solving, verbalization of thought processes, guided practice, extensive and corrective feedback, and frequent cumulative review.

In relation to Curriculum Focal Point 2, explicit instruction may help students conceptually understand decimals and fractions by providing a clearly articulated sequence of examples and non-examples as well as a strong vocabulary of mathematically precise terms. With explicit instruction, students develop procedural understanding by having clearly identified steps or procedures to follow. As students acquire experience with using procedures for different problem types, they are able to solve more complex problems with ease. Intervention instruction should also incorporate having students draw models of the mathematical concepts or processes.

Strategies Identified to Change Student Outcomes

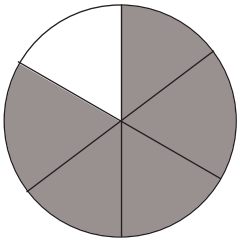
In connection with IES Recommendation 3 and Curriculum Focal Point 2 (CFP 2), intervention instruction should often include, but not be limited to, the following:

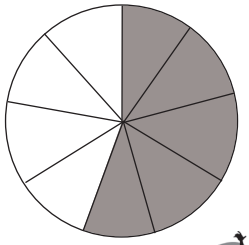
1. *Explaining the link between fractions and decimals* – In order to see the connections between fractions and decimals as different representations of rational numbers, students benefit from clear models, such as the example here.

Model	Fraction	Decimal
	How many boxes filled? *4* How many boxes total? *10*	0.4
	How many boxes filled? *7* How many boxes total? *10*	0.7
	How many boxes filled? *3* How many boxes total? *10*	0.3

   Moreover, when comparing or converting between representations, students may benefit from step-by-step approaches to solving problems. For example, teachers may create checklists or design think-aloud questions for students to use as they work through the problem.

2. *Using models to support students' understanding of rational numbers* – This is especially useful when students are developing conceptual understanding of the different magnitudes of fractions. When teachers draw models of these fractions and teach students how to draw their own models, students learn to visualize fractional quantities. For example: *In Sarah's reading class, ⅚ of the students are girls. In her math class, ⅔ of the students are girls. Which class has a greater fraction of girls?* In the circles here, it becomes easily apparent that ⅚ is greater than ⅔.





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3. *Teaching strategies to compare numbers* – When comparing decimals, students should first compare whole number amounts by using strategies they previously learned.
If the whole numbers are equal, students should then look at the decimal amounts. Students will compare the numbers from left to right, starting with the largest place value. For example, if students had to compare 12.47 and 12.39, they could create a table like this:

	tens	ones	decimal	tenths	hundredths
12.47	1	2	.	4	7
12.39	1	2	.	3	9

It is not necessary to compare the hundredths place in this example because it is clear from looking at the tenths place that 12.4 is larger than 12.3. Students may benefit from comparing decimals with a place value chart in which they can write the two numbers they are comparing one above the other.

4. *Teaching the place value of zero* – Students should be taught that adding zeros to the right of a decimal does not change its value. Students should recognize that 0.1 is equivalent to 0.10 and 0.100. This knowledge becomes important when operating with decimals in future math studies.

Please see the IES Recommendations and Curriculum Focal Points documents for further information.

References

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Thompson, Lauren and Linda S. Wingerter. *One Riddle, One Answer.* New York: Scholastic, 2001.